$$\Delta_{SRA} = U_{SRA} + Mg\,K_{SS}\cos L\cos A_T + Mg\,K_{SI}(\cos L\sin A_T\cos\beta - \sin L\sin\beta) + Mg\,K_{SO}(\cos L\sin A_T\sin\beta + \sin L\cos\beta)$$
$$\Delta_{IA} = U_{IA} + Mg\,K_{IS}\cos L\cos A_T + Mg\,K_{II}(\cos L\sin A_T\cos\beta - \sin L\sin\beta) + Mg\,K_{IO}(\cos L\sin A_T\sin\beta + \sin L\cos\beta) \quad (12)$$

From the preceding, the unbalance torque about OA is derived:

$$T_{OA} = R + \Delta_{SRA}F_{IA} - \Delta_{IA}F_{SRA} = R - Mg\,U_{SRA}\sin L\sin\beta - \tfrac{1}{2}M^2g^2K_{IS}\cos^2L + \tfrac{1}{4}M^2g^2K_{SO}(\cos^2L - 2\sin^2L)\sin 2\beta + \tfrac{1}{2}M^2g^2K_{SI}(\cos^2L\cos^2\beta + 2\sin^2L\sin^2\beta) + [Mg\,U_{SRA}\cos L\cos\beta - \tfrac{1}{2}M^2g^2K_{SI}\sin 2L\sin 2\beta + \tfrac{1}{2}M^2g^2K_{SO}\sin 2L\cos 2\beta]\sin A_T + [-Mg\,U_{IA}\cos L + \tfrac{1}{2}M^2g^2(K_{II} - K_{SS})\sin 2L\sin\beta - \tfrac{1}{2}M^2g^2K_{IO}\sin 2L\cos\beta]\cos A_T - [\tfrac{1}{2}M^2g^2(K_{II} - K_{SS})\cos^2L\cos\beta + \tfrac{1}{2}M^2g^2K_{IO}\cos^2L\sin\beta]\sin 2A_T - [\tfrac{1}{2}M^2g^2K_{SI}\cos^2L\cos^2\beta + \tfrac{1}{2}M^2g^2K_{IS}\cos^2L + \tfrac{1}{2}M^2g^2K_{SO}\cos^2L\sin 2\beta]\cos 2A_T \quad (13)$$

Once again the torque has been expressed in harmonics of A_T.

III. Summary

The results derived in the preceding section are useful for analysis of gyroscope test data in an attempt to improve compensation for elastic deflections. By means of appropriate tests, this provides a possible way of identifying and evaluating cross compliances. For example, a set of tests may be made with different values of β in case 1. For each value of β, a harmonic analysis will provide a coefficient of each harmonic of ϕ. Then the coefficients themselves may be analyzed by harmonic analysis in terms of β to evaluate the cross compliances. The accuracy of the analysis will depend strongly on the amount and nature of the interference or "noise" in the original data.

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Energy Separation in Laminar Vortex-Type Slip Flow

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Nomenclature

- c_p = specific heat at constant pressure
- d = tube diameter, $2r_0$
- k = thermal conductivity
- p = static pressure
- Pr = Prandtl number, $\mu c_p/k$
- q = heat transfer to the wall
- r = radial coordinate for tube; r_0 is tube radius
- T = temperature; T_w is wall temperature; T_g is gas temperature adjacent to wall; T_t is total temperature; $T_{t,g}$ is total temperature of gas adjacent to wall
- u = axial velocity; $\bar{u}$ is average velocity; u_c is centerline velocity; u_s is slip velocity
- v = tangential velocity; v_0 is velocity at radius r_0; v_s is slip velocity
- x = axial coordinate for tube
- η = dimensionless coordinate, r/r_0
- λ = dimensionless velocity, $u_s/\bar{u}$
- μ = absolute viscosity
- ξ_v = velocity slip coefficient
- ρ = density

Received January 31, 1963.

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THIS note is concerned with the effects of low-density phenomena on energy separation for laminar fluid flow in an insulated, rotating circular tube. Specifically, consideration is given to the slip-flow regime, wherein velocity and temperature discontinuities occur at the tube wall. It is felt that this study will provide some insight into the energy separation characteristics of a vortex tube operating under slip-flow conditions. Incompressible flow is assumed; in addition, it is assumed that the velocity-slip coefficient does not depend on the direction of the flow with respect to the surface, so that the single coefficient ξ_v suffices.

The appropriate forms of the Navier-Stokes equations (assumed to govern the flow) are

$$(\mu/r)(\partial/\partial r)[r(\partial u/\partial r)] = \partial p/\partial x \quad (1)$$

$$(\partial/\partial r)[(1/r)(\partial/\partial r)(rv)] = 0 \quad (2)$$

The boundary conditions imposed on u and v are[1]

$$\begin{aligned} u &\equiv u_s \equiv -\xi_v(du/dr)_{r=r_0} && \text{at } r = r_0 \\ du/dr &= 0 && \text{at } r = 0 \\ v &= v_0 - v_s \equiv v_0 - \xi_v(dv/dr)_{r=r_0} && \text{at } r = r_0 \\ v &= 0 && \text{at } r = 0 \end{aligned} \quad (3)$$

The velocity components as obtained by solution of these equations are

$$\begin{aligned} u &= 2\bar{u}[1 - \eta^2 + 4(\xi_v/d)]/[1 + 8(\xi_v/d)] \\ v &= v_0\eta/[1 + 2(\xi_v/d)] \end{aligned} \quad (4)$$

where $\eta \equiv r/r_0$. The energy equation for this case is

$$\rho c_p u(\partial T/\partial x) = (kr)(\partial/\partial r)[r\partial T/\partial r] + \mu(du/dr)^2 \quad (5)$$

subject to the boundary condition[2]

$$q = [k(\partial T/\partial r) + \mu u(\partial u/\partial r)]_{r=r_0} \equiv 0 \quad \text{(insulated wall)} \quad (6)$$

(The tangential flow produces no additional shear stresses.)

Equations (5) and (6) may be rewritten in the form[3]

$$\rho c_p u(\partial/\partial x)[T + (Pr/c_p)(u^2/2)] = (8\mu\bar{u}/r_0^2)(1 - \lambda)u + (kr)(\partial/\partial r)\{r(\partial/\partial r)[T + (Pr/c_p)(u^2/2)]\} \quad (7)$$

$$q = k(\partial/\partial r)[T + (Pr/c_p)(u^2/2)]_{r=r_0} \equiv 0 \quad (8)$$

Thus, at any given axial location, Eqs. (7) and (8) are satisfied by the solution

$$T + (Pr/c_p)(u^2/2) \equiv \text{const} \equiv T_g + (Pr/c_p)(u_s^2/2) \quad (9)$$

If the static temperature T is converted to the total temperature $[T_t \equiv T + (u^2 + v^2)/2c_p]$ and is made dimensionless through the use of the centerline axial velocity u_c, one obtains, after algebraic manipulation, the following expression:

$$(T_t - T_{t,g})/(u_c^2/2c_p) = (1 - Pr) \times \{[(2 - \lambda) - 2(1 - \lambda)\eta^2]^2 - \lambda^2\}/(2 - \lambda)^2 - 16(1 - \lambda)^2(v_0/u_c)^2(1 - \eta^2)/(4 - 3\lambda)^2 \quad (10)$$

The result is shown in Fig. 1 for several values of the parameters (v_0/u_c) and $(u_s/\bar{u})$ and for a Prandtl number of 0.7.

Inspection of this figure reveals the effect of the rarefaction. With continuum flow ($\lambda = 0$), a considerable variation in total temperature is realized for Poiseuille flow through a rotating tube, the effect becoming larger with in-

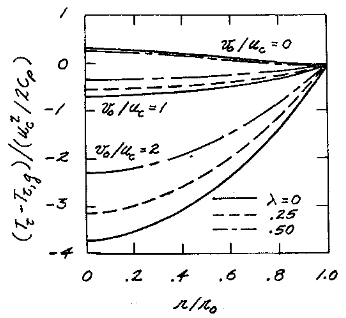

Fig. 1 Total temperature distribution for laminar slip flow through a rotating circular tube (Pr = 0.7)

creasing rotation;[4] however, with rarefaction or velocity jump ($\lambda \neq 0$), a diminution of the total temperature variation below the continuum flow value is experienced, and this decrease is accentuated with increasing slip. This effect of velocity jump on total temperature difference was observed earlier.[5]

The foregoing analysis suggests that a vortex tube using a slightly rarefied gas in which laminar slip flow occurs will exhibit a smaller energy separation effect than observed under laminar continuum flow conditions. A more detailed analytical study as well as an experimental study of vortex flows of a rarefied gas should prove illuminating.

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More on the Effectiveness Concept in Mass-Transfer Cooling

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Nomenclature

B_h = blowing rate for energy transfer $\equiv [(\rho v)_w/(\rho u)_\infty] \times (1/C_{h0})$
B_s = blowing rate based on sublayer thickness [cf., Eq. (7)]
$C_f/2$ = $\frac{1}{2} \times$ friction coefficient $\equiv \tau_w/\rho_\infty u_\infty^2$
C_h = Stanton number $\equiv [k_w(\partial T/\partial y)_w/(\rho u c_p)_\infty (T_r - T_w)]$
c_p = specific heat at constant pressure
k = thermal conductivity
Pr = Prandtl number $\equiv c_p\mu/k$
r = recovery factor $\equiv [2c_{p\infty}(T_r - T_\infty)/u_\infty^2]$
R = effectiveness $\equiv (T_w - T_c)/(T_{r0} - T_c)$
Re = Reynolds number $\equiv \rho u_\infty x/\mu$
T = temperature
T_c = temperature of coolant [cf., Eq. (1)]
T_r = temperature of wall for case in which temperature gradient vanishes at wall
u = velocity parallel to wall
v = velocity normal to wall
x = coordinate parallel to wall
y = coordinate normal to wall
μ = dynamic viscosity coefficient
ρ = density
τ = shearing stress

Superscripts

c = mixture component(s) (e.g., coolant) added at wall
$*$ = variable evaluated at reference state

Subscripts

s = sublayer boundary
w = wall
0 = limiting value as blowing rate approaches zero
∞ = outer edge of boundary layer

Received January 17, 1963; revision received February 8, 1963. The authors would like to acknowledge that computations suggested by E. R. Bartle led to the explanation of the apparent Reynolds number independence of the correlation presented by Bartle and Leadon.

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The effectiveness R was used recently by Bartle and Leadon in heat transfer correlations for nitrogen[1] and foreign gas[2] injections into turbulent air streams. They report[2] that "the effectiveness is found to represent the data well for a wide variety of coolant gases, Mach numbers 2 and 3.2, and small Reynolds number variations, when it is considered to a function only of $B_h c_p^c/c_{p\infty}$." (Nomenclature of the present paper is used.) Tewfik[3] points out some limitations of this parameter and concludes that "it is not of much use in correlating the reduction in heat transfer with injection, except perhaps when the wall temperature is much different from adiabatic, or in the special case of air injection in low-speed flow." The purpose of the present note is to examine analytically the dependence of the effectiveness upon temperatures, blowing rate, Mach number, and Reynolds number.

The heat-balance equation

$$C_h^*\rho^*u_\infty c_p^* (T_r - T_w) = (\rho v c_p^c)_w (T_w - T_c) \qquad (1)$$

may be rearranged[4] to obtain

$$R = \left[1 + \frac{(\rho v c_p^c)_w}{\rho^* u_\infty c_p^*} \frac{1}{C_h^*} \frac{T_{r0} - T_w}{T_r - T_w}\right]^{-1} \qquad (2)$$

For turbulent Prandtl number equal to unity, Knuth and Dershin[4] write, as an extension of the Reynolds analogy to the case of mass addition at the wall,

$$1 + \frac{(\rho v c_p^c)_w}{\rho^* u_\infty c_p^*} \frac{1}{C_h^*} = \left[\left(1 + \frac{(\rho v)_w}{\rho^* u_\infty} \frac{2}{C_f^*} \frac{u_s}{u_\infty}\right)^{Pr^{*-1}} \times \left(1 + \frac{(\rho v)_w}{\rho^* u_\infty} \frac{2}{C_f^*}\right)\right]^{c_p^c/c_p^*} \qquad (3)$$

Substituting from Eq. (3) into Eq. (2),

$$R = \left\langle 1 + \left\{\left[\left(1 + \frac{(\rho v)_w}{\rho^* u_\infty} \frac{2}{C_f^*} \frac{u_s}{u_\infty}\right)^{Pr^{*-1}} \times \left(1 + \frac{(\rho v)_w}{\rho^* u_\infty} \frac{2}{C_f^*}\right)\right]^{c_p^c/c_p^*} - 1\right\} \frac{T_{r0} - T_w}{T_r - T_w}\right\rangle^{-1} \qquad (4)$$

This analytical result is to be compared with the empirical result

$$R = [1 + \tfrac{1}{3} B_h (c_p^c/c_{p\infty})]^{-3} \qquad (5)$$

presented by Bartle and Leadon.[2] One might predict, using Eq. (4), values of R for the test conditions of Ref. 1 and compare these predicted values with the measured values. (Attempts to predict for the test conditions of Ref. 2 are held in abeyance until a reference-composition expression for turbulent flows is established.) Such predictions are more valuable if they can be made knowing only the design conditions